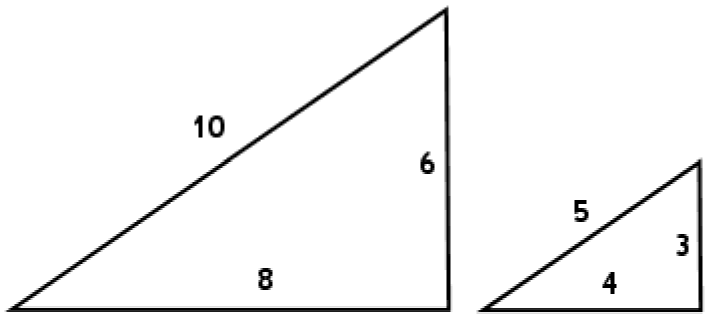
Basic Trigonometric Ratios

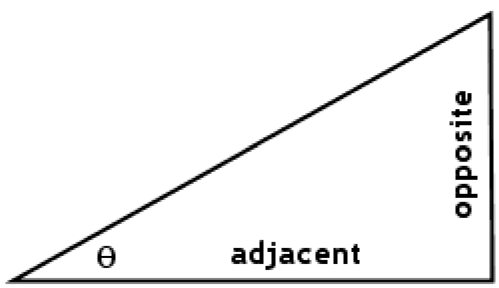
Right triangles are nice and neat, with their side lengths obeying the Pythagorean Theorem. Any two right triangles with the same two non-right angles are "similar", in the technical sense that their corresponding sides are in proportion. For instance, the following two triangles (*not* drawn to scale) have all the same angles, so they are similar, and the corresponding pairs of their sides are in proportion:



ratios of corresponding sides: $10/5 = 8/4 = 6/3 = 2$
ratio of hypotenuse to base: $10/8 = 5/4 = 1.25$
ratio of hypotenuse to height: $10/6 = 5/3 = 1.666...$
ratio of height to base: $6/8 = 3/4 = 0.75$

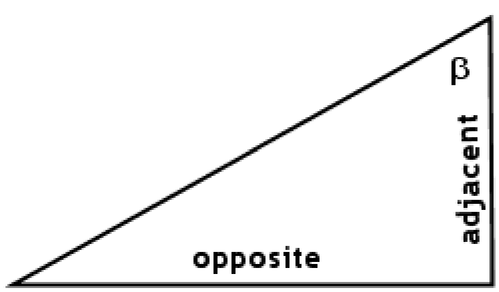
Around the fourth or fifth century AD, somebody very clever living in or around India noticed these consistency of the proportionalities of right triangles with the same sized base angles, and started working on tables of ratios corresponding to those non-right angles. There would be one set of ratios for the one-degree angle in a 1-89-90 triangle, another set of ratios for the two-degree angle in a 2-88-90 triangle, and so forth. These ratios are called the "trigonometric" ratios for a right triangle.

Given a right triangle with a non-right angle designated as θ ("THAY-tuh"), we can label the hypotenuse (always the side opposite the right angle) and then label the other two sides "with respect to θ" (that is, in relation to the non-right angle θ that we're working with).



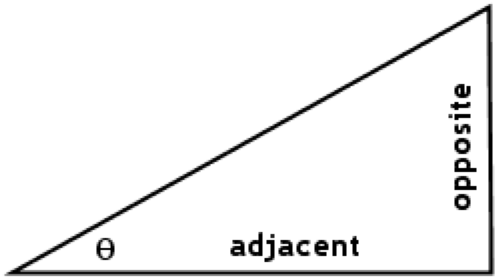
The side opposite the angle θ is the "opposite" side, and the other side, being "next" to the angle (but not being the hypotenuse) is the "adjacent" side.

For the same triangle, if we called the third angle β ("BAY-tuh"), the labeling would be as shown:



As you can see, the labels "opposite" and "adjacent" are relative to the angle in question.

Let's return to the first labeling:



There are six ways to form ratios of the three sides of this triangle. I'll shorten the names from "hypotenuse", "adjacent", and "opposite" to "hyp", "adj", and "opp":

name	ratio	notation	name	ratio	notation
	opp/hyp			hyp/opp	
	adj/hyp			hyp/adj	
	opp/adj			adj/opp	

Each of these ratios has a name:

name	ratio	notation	name	ratio	notation
sine	opp/hyp		cosecant	hyp/opp	
cosine	adj/hyp		secant	hyp/adj	
tangent	opp/adj		cotangent	adj/opp	

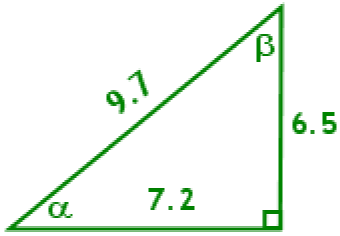
...and each of these names has an abbreviated notation, specifying the angle you're working with:

name	ratio	notation	name	ratio	notation
sine	opp/hyp	$sin(\theta)$	cosecant	hyp/opp	$csc(\theta)$
cosine	adj/hyp	$cos(\theta)$	secant	hyp/adj	$sec(\theta)$
tangent	opp/adj	$tan(\theta)$	cotangent	adj/opp	$cot(\theta)$

The ratios in the left-hand table are the "regular" trig ratios; the ones in the right-hand table are their reciprocals (that is, the inverted fractions). To remember the ratios for the regular trig functions, many students use the mnemonic SOH CAH TOA, pronounced "SOH-kuh-TOH-uh" (as though it's all one word). This mnemonic stands for:

- **S**ine is **O**pposite over **H**ypotenuse
- **C**osine is **A**djacent over **H**ypotenuse
- **T**angent is **O**pposite over **A**djacent

- **List the values of $sin(\alpha)$, $cos(\alpha)$, $sin(\beta)$, and $tan(\beta)$ for the triangle below, accurate to three decimal places:**

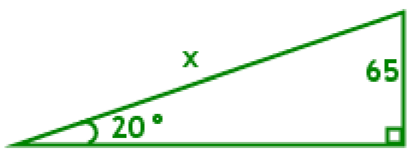


For either angle, the hypotenuse has length 9.7. For the angle α, "opposite" is 6.5 and "adjacent" is 7.2, so the sine of α will be 6.5/9.7 = 0.6701030928... and the cosine of α will be 7.2/9.7 = 0.7422680412.... For the angle β, "opposite" is 7.2 and "adjacent" is 6.5, so the sine of β will be 7.2/9.7 = 0.7422680412... and the tangent of β will be 7.2/6.5 = 1.107692308.... Rounding to three decimal places, I get:

$sin(\alpha) = 0.670$, $cos(\alpha) = 0.742$, $sin(\beta) = 0.742$, $tan(\beta) = 1.108$

Once you've memorized the trig ratios, you can start using them to find other values. You'll likely need to use a calculator. If your calculator does not have keys or menu options with "SIN", "COS", and "TAN", then now is the time to upgrade! Make sure you know how to use the calculator, too; the owner's manual should have clear instructions.

- **In the triangle shown below, find the value of x, accurate to three decimal places.**



They've given me an angle measure and the length of the side "opposite" this angle, and have asked me for the length of the hypotenuse. The sine ratio is "opposite over hypotenuse", so I can turn what they've given me into an equation:

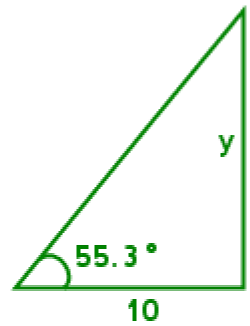
$sin(20°) = 65/x$
$x = 65/sin(20°)$

I have to plug this into my calculator to get the value of x: $x = 190.047286...$

$x = 190.047$

Note: If your calculator displayed a value of 71.19813587..., then check the "mode": your calculator is set to "radians" rather than to "degrees". You'll learn about radians later.

- **For the triangle shown, find the value of *y*, accurate to four decimal places.**



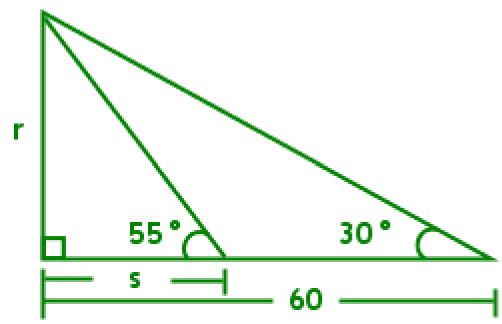
They've given me an angle, a value for "adjacent", and a variable for "opposite", so I can form an equation:

$$tan(55.3°) = y/10$$
$$10tan(55.3°) = y$$

Plugging this into my calculator, I get $y = 14.44183406....$

$y = 14.4418$

- **Find the angles and sides indicated by the letters in the diagram. Give each answer correct to the nearest whole number.**



At first, this looks fairly intimidating. But then I notice that, to find the length of the height r, I can use the base angle $30°$ and the full base length of 60, because $r/60$ is "opposite" over "adjacent", which is the tangent.

$$r/60 = tan(30°)$$
$$r = 60tan(30°) = 34.64101615...$$

I'm supposed to the nearest whole number, so $r = 35$.

Now that I have the value of r, I can use r and the other base angle, $55°$, to find the length of the other base, s, by using $r/s = tan(55°)$:

$$35/s = tan(55°)$$
$$35/tan(55°) = s = 24.50726384...$$

$r = 35, s = 25$

Note: Since the sine and cosine ratios involve dividing a leg (one of the shorter two sides) by the hypotenuse, the values will never be more than 1, because (some number) / (a bigger number) from a right triangle is always going to be smaller than 1. But you can have really wide and short or really tall and skinny right triangles, so "opposite" and "adjacent" can have very different values. This tells you that the tangent ratio, being (opposite) / (adjacent), can have very large and very small values, depending on the triangle.