

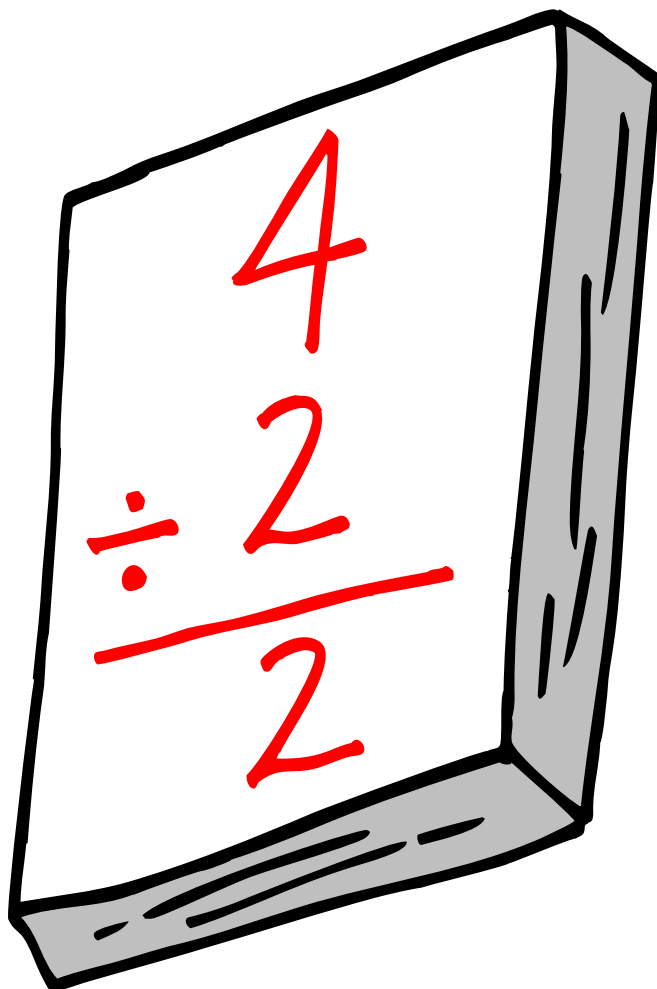


UNIVERSITY INTERSCHOLASTIC LEAGUE

Making a World of Difference

# Mathematics

Invitational A • 2017



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YOU ARE INSTRUCTED TO DO SO!

1. Evaluate:  $5.4 \div (\frac{3}{2})^{-1} - (1)^{-2} \times 3! + 4.5$

- (A)  $-3.9$       (B)  $\frac{1}{6}$       (C)  $\frac{3}{5}$       (D)  $2.1$       (E)  $6.6$

2. If  $\frac{3}{8}$  of A is 87.5% more than B, then A is what percent of B?

- (A) 5%      (B) 20%      (C) 50%      (D)  $233\frac{1}{3}\%$       (E) 500%

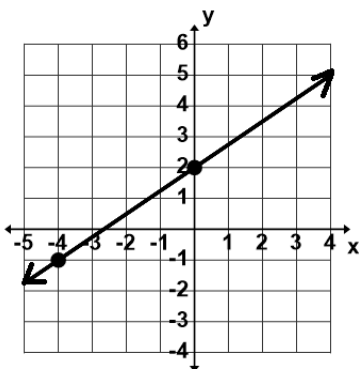
3. One billion two and three-fourth million five hundred six is added to six million fifty-four thousand three hundred twenty-one. How many digits in the sum are twos?

- (A) 1      (B) 2      (C) 3      (D) 4      (E) 5

4. Kookie Baykur baked some cookies. She took 20% of them to her grandmother. Then she ate 4 for lunch. After lunch she sold  $\frac{1}{2}$  of what was left at her school's bake sale. She had 6 left to share with her parents for after supper. How many cookies did she bake originally?

- (A) 18      (B) 20      (C) 24      (D) 30      (E) 36

5. A line parallel to the line shown containing the point (6, 3) contains which of the following points?



- (A)  $(-3, -4)$       (B)  $(-6, -6)$       (C)  $(0, -2)$       (D)  $(9, 7)$       (E)  $(10, 6)$

6. Let p and q be the roots of  $2x^2 + 3x - 5 = 0$ . Find  $p^3 + 3p^2q + 3pq^2 + q^3$ .

- (A) 15.625      (B) 6.5      (C) 3.625      (D)  $-1.125$       (E)  $-3.375$

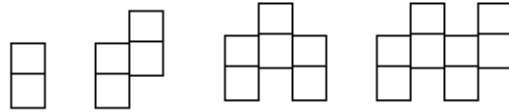
7. Phil D. Belly budgets \$53.00 per week for lunch. He spends \$7.00 each day that he goes to McDee's Grill and \$9.00 each day that he goes to Queen's Burger. How much more does he spend at McDee's Grill than at Queen's Burger during a 7 day week?

- (A) \$1.00      (B) \$15.00      (C) \$17.00      (D) \$33.00      (E) \$35.00

8.  $\angle A$  and  $\angle B$  are supplementary. If  $m \angle A = 3x + 4$  and  $m \angle B = 2x + 1$ , the measure of the larger angle is:

- (A)  $55^\circ$       (B)  $61^\circ$       (C)  $109^\circ$       (D)  $112^\circ$       (E)  $115^\circ$

9. The four shapes below are made up of 1 cm squares. If the pattern continues, find the perimeter of the shape consisting of 16 squares.



- (A) 26      (B) 30      (C) 32      (D) 34      (E) 40

10. M. T. Tank has a rectangular based water tank that is empty. The length of the tank is twice the width and the height is half of the width. How many gallons of water will he need to fill the tank if the height is 4 feet? (nearest gallon)

- (A) 3,830 gal      (B) 3,456 gal      (C) 3,192 gal      (D) 2,608 gal      (E) 2,095 gal

11. The point  $(3, -4)$  is rotated  $450^\circ$  clockwise about the origin. The coordinates of the point after the rotation is \_\_\_\_\_.

- (A)  $(-3, 4)$       (B)  $(4, -3)$       (C)  $(-3, -4)$       (D)  $(3, 4)$       (E)  $(-4, -3)$

12. If  $\frac{5x-2}{3x+1} + \frac{Ax-B}{x+4} = \frac{11x^2+5x-13}{3x^2+13x+4}$ , where A and B are constants, then  $A \times B$  equals:

- (A)  $-3$       (B)  $-1$       (C)  $3$       (D)  $7$       (E)  $10$

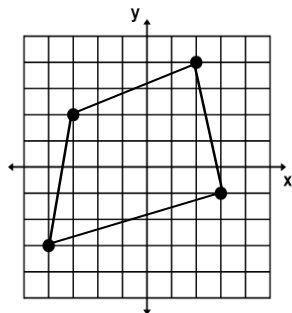
13. The roots of the equation  $2x^3 - x^2 - 5x - 2 = 0$  are  $-1$ ,  $2$ , and R. Find R.

- (A) 2.5      (B) 1      (C)  $-0.5$       (D)  $-1$       (E)  $-1.5$

14. Let  $A = \begin{bmatrix} 1 & 6 \\ -9 & -7 \end{bmatrix}$  and  $B = \begin{bmatrix} 2 & -1 \\ 0 & -7 \end{bmatrix}$ . Find  $|A - B|$ .

- (A)  $-15$       (B)  $-3$       (C)  $33$       (D)  $61$       (E)  $63$

15. Find the area of the quadrilateral. The coordinates of the vertices are integers.



- (A) 31.5 sq. units      (B) 30 sq. units      (C) 28.5 sq. units      (D) 28 sq. units      (E) 26 sq. units

16. Determine the frequency of  $f(x) = 3 + 5\sin[4\pi(x - 2)]$ .

- (A)  $\frac{1}{2}$       (B) 2      (C) 3      (D) 4      (E) 5

17. A plane is 120 miles north and 85 miles east of an airport. What bearing should the plane take to fly directly to the airport?

- (A)  $65^\circ$       (B)  $55^\circ$       (C)  $45^\circ$       (D)  $35^\circ$       (E)  $25^\circ$

18. Given the arithmetic sequence 15, a, b, 37, c, ..., find  $a + b + c$ .

- (A)  $96\frac{1}{3}$       (B) 114      (C)  $148\frac{2}{3}$       (D)  $81\frac{2}{3}$       (E)  $73\frac{1}{3}$

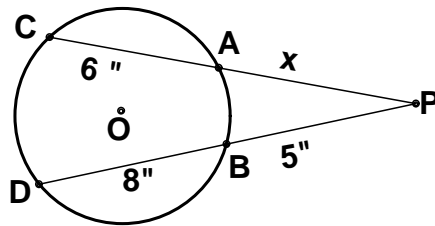
19. Find the remainder when  $x^3 + 2x^2 - 3x + 4$  is divided by  $x + 1$ .

- (A) 10      (B) 8      (C) 7      (D) 5      (E) 4

20. Find the eccentricity of the ellipse,  $16x^2 + 100y^2 = 1600$ . (nearest hundredth)

- (A) 0.87      (B) 0.90      (C) 0.92      (D) 0.95      (E) 0.98

21. Given the circle with center O shown. Find x. (nearest tenth).



- (A) 3.8      (B) 5.6      (C) 6.3      (D) 6.7      (E) 9.6

22. What is the sum of the digits in the tens place and the units place of  $7^{65}$ ?

- (A) 1      (B) 3      (C) 7      (D) 9      (E) 13

23. The function  $f(x) = x^2$  is concave up on which of the following open intervals?

- I. (0, 5)      II. (-5, 5)      III. (-5, 0)

- (A) I only      (B) II only      (C) III only      (D) I, II & III      (E) none of them

24. The graph of  $g(x) = (x^3 + 3x^2 + 3x + 1) \div (x^2 - 1)$  has vertical asymptote(s) at:

- (A)  $x = 1$       (B)  $x = -1$       (C)  $x = 1$  and  $-1$       (D)  $x = 0$       (E)  $g(x)$  has no vertical asymptotes

25. Let  $f''(x) = 18x + 4$ ,  $f'(-1) = 6$ , and  $f(1) = 6$ . Find  $f(-2)$ .

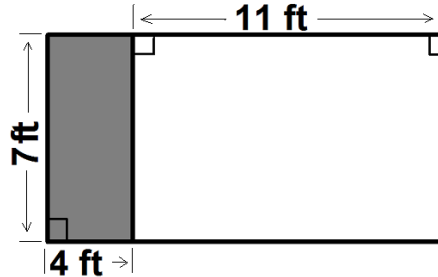
- (A) -34      (B) -32      (C) -18      (D) -12      (E) 12

26. Suppose A, B, and C are positive integers such that  $\frac{32}{5} = A + \frac{1}{B + \frac{1}{C+1}}$ .

The value of  $3A + 2B + 5C$  equals:

- (A) 9                      (B)  $9\frac{2}{5}$                       (C)  $13\frac{1}{2}$                       (D) 27                      (E) 37

27. Spud Pharmer's son, Tater, buried his daddy's shovel in their rectangular garden. What is the probability that it was buried in the shaded section shown? (nearest whole percent)



- (A) 7%                      (B) 20%                      (C) 25%                      (D) 27%                      (E) 36%

28. Lyn Koln flipped a penny four times and recorded the results. What are the odds of three or more consecutive heads occurring?

- (A)  $\frac{3}{16}$                       (B)  $\frac{1}{7}$                       (C)  $\frac{1}{8}$                       (D)  $\frac{5}{11}$                       (E)  $\frac{3}{13}$

29. Which of the following mathematicians are associated with for working with prime numbers?

- I. Eratosthenes of Cyrene      II. Sophie Germain      III. Marin Mersenne

- (A) I only                      (B) I & II                      (C) I & III                      (D) I, II & III                      (E) none of them

30. The number 13 is a member of which of the following sets of numbers?

- (A) abundant                      (E) vil                      (L)ucas                      (P)rimeval

- (A) L & P only                      (B) P only                      (C) E & L only                      (D) none of them                      (E) all of them

31. If 2 Babs equal 3 Bibs and 5 Bibs equal 7 Bobs, then how many Babs does it take to make 3 Bobs?

- (A)  $3\frac{1}{3}$                       (B)  $2\frac{5}{7}$                       (C)  $1\frac{3}{7}$                       (D)  $1\frac{1}{2}$                       (E)  $\frac{7}{10}$

32. Let U (universal set) = {0, 1, 2, 3, 4, 5, 6, 7, 8, 9}, P = {2, 3, 5, 7, 9}, and Q = {2, 1, 3, 4, 7}. Let R = (P ∩ Q)<sup>C</sup>. Set R contains how many distinct elements?

- (A) 3                      (B) 5                      (C) 7                      (D) 10                      (E) none

33. Soh Yung is 7 years older than her sister Tu Yung. In 3 years Soh will be twice as old as Tu. How old will Tu be in 5 years?

- (A) 4                      (B) 9                      (C) 11                      (D) 14                      (E) 16

34. Seven students in Miss Work's math class had test scores of 75, 83, 85, 92, 95, 98, and 100. Three of her students haven't take the test yet. What will the remaining three students have to average so that the entire class average is 88?

- (A) 84            (B) 85            (C) 86            (D) 87            (E) 88

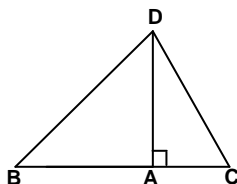
35. Rusty Yatt sailed his boat to Junk Yard Bay and back home. The trip took 12 hours going and 9 hours coming back. His average speed coming back was 20 kph. What was his average speed going?

- (A) 8 kph            (B) 11 kph            (C) 15 kph            (D) 21 kph            (E) 28 kph

36. Dee Deeler has a standard deck of cards consisting of 4 Aces, 12 face cards, and 36 number cards. No Joker is allowed. Dee wants to see how many 5 card hands he can create such that each hand has 1 Ace, 2 face cards, and 2 number cards. How many such hands can he make?

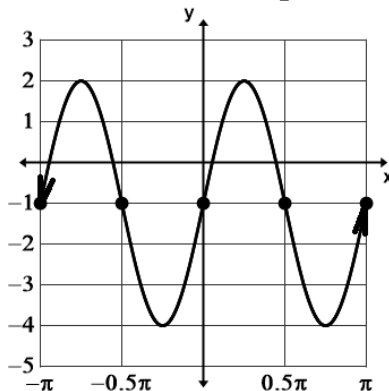
- (A) 166,320            (B) 260            (C) 8,640            (D) 1,728            (E) 665,280

37. Find the perimeter of  $\triangle BCD$  if  $AD = 3''$ ,  $m\angle ADB = 45^\circ$ , and  $m\angle ACD = 60^\circ$ . (nearest tenth)



- (A) 7.6"            (B) 8.2"            (C) 10.2"            (D) 12.4"            (E) not enough information given

38. The equation  $y =$  \_\_\_\_\_ will produce this graph.



- (A)  $3\cos(2x + \pi)$             (B)  $3\sin(2x) - 1$             (C)  $3\cos(2x + \pi) - 1$   
 (D)  $3\sin(2x - 1)$             (E)  $2 - 3\sin(x)$

39. The function  $f(x) = 3x^2 - 4x - 4$  crosses the x-axis at two points. Find the distance between the two points.

- (A)  $3\frac{1}{2}$  units            (B)  $1\frac{1}{3}$  units            (C) 4 units            (D)  $2\frac{2}{3}$  units            (E)  $1\frac{1}{2}$  units

40. The point of concurrency of the angle bisectors of a triangle is called the:
- (A) incenter (B) centroid (C) orthocenter (D) circumcenter (E) line of Euler
41. If  $a_1 = -1$ ,  $a_2 = -2$ ,  $a_3 = 3$ , and  $a_n = (a_{n-1})(a_{n-3}) - (a_{n-2})$ , where  $n > 3$  then  $a_6 = ?$
- (A)  $-2$  (B)  $-1$  (C)  $0$  (D)  $1$  (E)  $2$
42. Determine the range of  $(x) = 3 + 5\sin[4\pi(x - 2)]$ .
- (A)  $[-5, 5]$  (B)  $[-5, 8]$  (C)  $[-2, 8]$  (D)  $[-2, 5]$  (E)  $[-8, 8]$
43. Find the area (in square units) of the region bounded by  $y = -x^2$  and  $y = -4$ .
- (A)  $16$  (B)  $11\frac{1}{3}$  (C)  $10\frac{2}{3}$  (D)  $8$  (E)  $5\frac{1}{3}$
44.  $\frac{1+4+9+16+\dots+64+81}{1+3+6+10+\dots+36+45} = \underline{\hspace{2cm}}$ .
- (A)  $1\frac{4}{5}$  (B)  $1\frac{9}{11}$  (C)  $1\frac{64}{81}$  (D)  $1\frac{74}{101}$  (E)  $1\frac{8}{11}$
45. Let  $f_0 = 0$ ,  $f_1 = 1$ ,  $f_2 = 1$ ,  $f_3 = 2$ ,  $f_4 = 3$ , ... be the terms of the Fibonacci sequence. If  $f_n = 121,393$  then  $n$  is:
- (A)  $20$  (B)  $22$  (C)  $24$  (D)  $26$  (E)  $28$
46. Willie Pikette is going to randomly pick two different numbers from the set  $\{2, 1, 3, 4, 7, 11\}$ . What is the probability that the sum of the two numbers he picks will be a prime number?
- (A)  $20\%$  (B)  $26\frac{2}{3}\%$  (C)  $33\frac{1}{3}\%$  (D)  $40\%$  (E)  $53\frac{1}{3}\%$
47. The function  $f$  is defined by  $f(x) = 2 + \ln(x + 3)$ . The inverse function of  $f$  is  $f^{-1}(x) = ?$
- (A)  $(2 + \ln(x + 3))^{-1}$  (B)  $\ln(x - 2)$  (C)  $e^{(x+2)} - 3$   
(D)  $e^{(x-2)} - 3$  (E)  $-(2 + \ln(x + 3))^{-1}$
48. Let  $f(x) = \frac{x^3 - 3x^2}{x^2 - 1}$  and  $s(x)$  be the slant asymptote of  $f$ . Find the value of  $s(4)$ .
- (A)  $1\frac{1}{15}$  (B)  $-1$  (C)  $\frac{15}{16}$  (D)  $1$  (E)  $7$
49. Alice, Bob, Charlie, Dan, and Edith sit randomly in a row of five chairs. What is the probability that Alice and Edith sit next to each other? (nearest percent)
- (A)  $3\%$  (B)  $7\%$  (C)  $20\%$  (D)  $35\%$  (E)  $40\%$
50. Find the slope of the line tangent to the curve  $y = x^2 - 3x + 5$  at  $(3, 5)$ .
- (A)  $2$  (B)  $3$  (C)  $5$  (D)  $6$  (E)  $10$

51. If the three numbers 78, 169, and 246 are each divided by the number D, each of their quotients will have the same remainder R. Find R.
- (A) 7                      (B) 5                      (C) 3                      (D) 2                      (E) 1
52. Let  $f(x) = x^3 + 2x^2 - 4x$ . Find the sum of the x-values of the critical points of the function.
- (A) 2                      (B) 1                      (C)  $-\frac{2}{3}$                       (D)  $-1\frac{1}{3}$                       (E)  $-2\frac{2}{3}$
53. Let  $g(x) = x^2 + 2x + 1$ . Find k if  $g(k + 1) - g(k) = 7$ .
- (A) -2                      (B) -1                      (C) 0                      (D) 1                      (E) 2
54. Let  $f_0 = 0, f_1 = 1, f_2 = 1, f_3 = 2, f_4 = 3, \dots$  be the terms of the Fibonacci sequence. How many digits are in  $f_{21}$ ?
- (A) 3                      (B) 4                      (C) 5                      (D) 6                      (E) 7
55.  $14_5 + 32_5 \times 23_5 = \underline{\hspace{2cm}}_5$
- (A) 1410                      (B) 1300                      (C) 1113                      (D) 2314                      (E) 2323
56. If  $15x^2 + cx - 12 = (5x + a)(bx - 4)$  then  $a + b + c = \underline{\hspace{2cm}}$ .
- (A) -5                      (B) -2                      (C) 3                      (D) 6                      (E) 17
57. Let  $f(x) = 5x - 2, g(x) = x + 4, h(x) = 3x + 1$ , and  $g(f(h(x))) = 10$ . Find x.
- (A)  $-\frac{13}{15}$                       (B)  $-\frac{1}{2}$                       (C)  $\frac{1}{5}$                       (D)  $\frac{7}{9}$                       (E)  $1\frac{2}{15}$
58. How many 3-digit numbers can be made using the digits 2, 1, 3, 4, and 7?
- (A) 64                      (B) 60                      (C) 32                      (D) 30                      (E) 15
59. The sequence 6, p, q, 1.5 is a harmonic sequence. Find the value of  $p + q$ .
- (A)  $\frac{5}{6}$                       (B)  $1\frac{1}{5}$                       (C)  $3\frac{3}{4}$                       (D) 5                      (E) 7.5
60. A right triangle,  $\triangle ABC$ , with leg lengths 15" and 20" and the right angle at vertex B is congruent to right triangle,  $\triangle BDE$ , with the right angle at vertex D. Point C lies on segment BD and points A and E are on the same side of segment BD. Find the distance between points A and E. (nearest eighth of an inch).
- (A)  $20\frac{5}{8}$ "                      (B)  $20\frac{1}{4}$ "                      (C)  $19\frac{7}{8}$ "                      (D)  $18\frac{3}{8}$ "                      (E)  $17\frac{1}{2}$ "